

Year 13 Finals 2023

1. Find the value of $a + b$ if two real numbers a and b satisfy $a(1 + 2i) + b(2 - i) = 8 + 6i$

2. A number $c + di$ has a magnitude of 15. If d is positive and 75% of c , then find c .

3. What is the minimum value of $y = 2\sin x - \sqrt{12}\cos x$?

4. What is $\text{Im}(z)$, the imaginary part of z , if $z - \bar{z} = 10i$?

5. Find the value of $\sqrt{\frac{(2 - 2i)(5 + i)}{3 - 2i}}$

6. Let $f(x)=\sqrt{x+3}$ and $g(x)=\sqrt{2-x}$

Give the domain of $(f + g)(x)$

7. Let $f(x) = x^2$ and $g(x)=\sqrt{9-x^2}$

What is the domain of $f \circ g(x)$?

8. Evaluate $\sum_{n=1}^{2023} i^n$

9. A term in the expansion of $\left(2x^3 - \frac{1}{x}\right)^{12}$ is $\binom{12}{k}(2x^3)^k\left(\frac{-1}{x}\right)^{12-k}$

Find the value of k if it's a constant term.

10. What is a directional vector of the line $\frac{3-x}{3} = \frac{2y-6}{4} = 3-z$ in terms of i , j and k ?

11. The graph of $y = \frac{2x^2 - 3x - 2}{x - 3}$ has slant (oblique) asymptote $y = ax + b$ and vertical asymptote $x = c$. Find the value of $ac + b$

12. Find the exact value of $\sec(\sin^{-1} \frac{1}{4})$

13. A car moves in a straight road. Its displacement s in m is given below where t is the time in seconds is $s = 3t^2 - 4t + 8$

Find the acceleration of the car after 2 seconds.

14. The sequence $\langle a_n \rangle = \frac{kn - 1}{3n + 4}$ converges to 5

Find the value of k .

15. If $\frac{3-x}{x^2-1} = \frac{A}{x+1} + \frac{B}{x-1}$ find the value of $A + B$.

16. Find the area between $y=x$ and $y=x^2-2$

17. Solve the equation $2\cos^2 x + 3 = 13 + 8\cos x$ for $0 \leq x \leq 360^\circ$

Give your answer in degrees.

18. Given that $\int e^x dx = e^x$, evaluate $\int_{-1}^1 e^{3x} - \frac{1}{e^{3x}} dx$

19. A stone dropped into a pond, creates a circular ripple. The diameter of the ripple increases at a constant rate of 10 cm/s. At what rate, in cm^2/s , is the area enclosed within the ripple increasing after 3 seconds? Give your answer in terms of π .

20. Determine the interval on which $f(x)=4x^2(x-3)-10$ is **concave up**.
